A quark-meson coupling model with short-range quark-quark correlations

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Abstract

Short-range quark-quark correlations are introduced into the quark-meson coupling (QMC) model in a simple way. The effect of these correlations on the structure of the nucleon in dense nuclear matter is studied. We find that the short-range correlations may serve to reduce a serious problem associated with the modified quark-meson coupling model (within which the bag constant is allowed to decrease with increasing density), namely the tendency for the size of the bound nucleon to increase rapidly as the density rises. We also find that, with the addition of correlations, both QMC and modified QMC are consistent with the phenomenological equation of state at high density.

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About a decade ago, Guichon [1] proposed a relativistic quark model for nuclear matter, where it consists of non-overlapping nucleon bags bound by the self-consistent exchange of scalar (σ) and vector (ω) mesons in mean-field approximation (MFA). This model has been further developed as the quark-meson coupling (QMC) model, and applied to various phenomena in nuclear physics (for recent reviews, see Ref.[2]). Recently, Jin and Jennings [3] have proposed an alternative version of QMC (called the modified QMC, MQMC), where the bag constant is allowed to decrease as a function of density.

So far, the use of the QMC model has been limited to the region of small to moderate densities, because it has been assumed that the nucleon bags do not overlap. It is therefore of great interest to explore ways to extend the model to include short-range quark-quark correlations, which may occur when nucleon bags overlap at high density. In this paper we will introduce these short-range correlations in a very simple way, and calculate their effect on the quark structure of the nucleon in medium. We refer to this model as the quark-meson coupling (or modified quark-meson coupling) model with short-range correlations (QMCs (or MQMCs)).

Let us consider uniformly distributed (iso-symmetric) nuclear matter with density ρ_B. At high density the nucleon bags start to overlap with each other, and a quark in one nucleon may interact with quarks in other nucleons in the overlapping region. Since the interaction between the quarks is short range, it seems reasonable to treat it in terms of contact interactions. An additional interaction term of the form, $\mathcal{L}_{int} \sim \sum_{i \neq j} \bar{\psi}_q(i) \Gamma_\alpha \psi_q(i) \bar{\psi}_q(j) \Gamma^\alpha \psi_q(j)$, may then be added to the original QMC Lagrangian density [2]. Here $\psi_q(i)$ is a quark field in the i-th nucleon and Γ_α stands for $1, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu$ or $\sigma_{\mu\nu}$ (with or without the isospin and color generators). (For the present we consider only u and d quarks.) In MFA most of these terms vanish in a static, spin saturated, uniform system because of rotational symmetry, parity etc. We shall retain only the dominant MFA contributions, namely the scalar- and vector-type interactions: $\Gamma_\alpha = 1$ and γ_0.

Next we consider the probability for the nucleon bags to overlap, using a simple geometrical approach. Let us first consider a collection of rigid balls, with a radius R_c. In

the close-packed structure of nuclear matter we find that the effective volume per ball is given by $V_c = 4\sqrt{2}R_c^3$. The corresponding density, ρ_c, is then given by the inverse of V_c, and hence the radius of the rigid ball is related to the density as $R_c = 1/(4\sqrt{2}\rho_c)^{1/3}$. Returning to our problem, we see that for a given nuclear density ρ_B, if the nucleon bag radius, R, which is given by solving the nuclear matter problem self-consistently, is larger than $R_c (= 1/(4\sqrt{2}\rho_B)^{1/3})$ the nucleons will overlap. If $R \leq R_c$, there is no overlap. Of course, one could build a more sophisticated model, allowing for nucleon motion and nucleon-nucleon correlations [4], but we believe that the present model is sufficient for an initial investigation.

Now consider two nucleon bags separated by a distance d in nuclear matter. They will overlap for $d < 2R$ and the common volume is then given by $V_{ov} = V_N(1-3y/4+y^3/16)$ [5], where V_N is the nucleon volume ($= 4\pi R^3/3$) and $y = d/R$. It is natural to choose the probability of overlap, p, to be proportional to V_{ov}/V_N:

$$p(y) \propto 1 - \frac{3y}{4} + \frac{y^3}{16}. \tag{1}$$

Of course, this choice is quite model-dependent. In principle we could use an arbitrary, smooth function, which goes to unity at $y = 0$ and zero beyond $y = 2$ and which respects the three dimensional geometry of this problem. In this exploratory study, we take this simple form as an example.

In mean-field approximation the Dirac equation for a quark field in a nucleon bag is given by

$$[i\gamma \cdot \partial - (m_q - g_\sigma^q \sigma + f_s^q \langle \bar{\psi}_q \psi_q \rangle) - (g_\omega^q \omega + f_v^q \langle \psi_q^\dagger \psi_q \rangle)\gamma_0]\psi_q = 0, \tag{2}$$

where m_q is the bare quark mass, σ and ω are the mean-field values of the σ and ω mesons and g_σ^q and g_ω^q are, respectively, the σ- and ω-quark coupling constants in the usual QMC model [2]. The new coupling constants, $f_{s(v)}^q$, have been introduced for the scalar (vector)-type short-range correlations, and are given by (see Eq.(1))

$$f_{s(v)}^q = \frac{\bar{f}_{s(v)}^q}{M^2} \times (1 - \frac{3y}{4} + \frac{y^3}{16})\theta(y)\theta(2 - y). \tag{3}$$

We have also taken $y = d/R$, with d the average distance between two neighbouring nucleons at a given nuclear density ρ_B – i.e., as explained above, $d = 2R_c = 2/(4\sqrt{2}\rho_B)^{1/3}$. Note that since the coupling constants have dimension of (energy)$^{-2}$ we introduce new, dimensionless coupling constants, $\bar{f}_s^q$ and $\bar{f}_v^q$ by dividing by the free nucleon mass ($M = 939$ MeV) squared. (If the coupling strength is positive the correlation gives a repulsive force.) In Eq.(2), $\langle \bar{\psi}_q \psi_q \rangle$ and $\langle \psi_q^\dagger \psi_q \rangle$ are, respectively, the average values of the quark scalar density and quark density with respect to the nuclear ground state, which are approximately given by the values at the center of the nucleon in local density approximation [6] (we will revisit this later).

Now we can solve the Dirac equation, Eq.(2), as in the usual QMC, with the effective quark mass

$$m_q^\star = m_q - g_\sigma^q \sigma + f_s^q \langle \bar{\psi}_q \psi_q \rangle, \tag{4}$$

instead of the bare quark mass. The Lorentz vector interaction shifts the nucleon energy in the medium [7]:

$$\epsilon(\vec{k}) = \sqrt{M^{\star 2} + \vec{k}^2} + 3(g_\omega^q \omega + f_v^q \langle \psi_q^\dagger \psi_q \rangle), \tag{5}$$

where $M^\star$ is the effective nucleon mass, which is given by the usual bag energy

$$M^\star = \frac{3\Omega - z}{R} + \frac{4}{3}\pi B R^3. \tag{6}$$

Here B and z are respectively the bag constant and usual parameter which accounts for zero-point motion and gluon fluctuations [7]. The quark energy, Ω (in units of $1/R$), is defined by $\sqrt{x^2 + (Rm_q^\star)^2}$, where x is the lowest eigenvalue of the quark, which is given by the usual boundary condition at the bag surface [7].

The total energy per nucleon at density ρ_B is then expressed as

$$
\begin{aligned}
E_{tot} &= \frac{4}{(2\pi)^3 \rho_B} \int^{k_F} d\vec{k} \sqrt{M^{\star 2} + \vec{k}^2} + 3(g_\omega^q \omega + f_v^q \langle \psi_q^\dagger \psi_q \rangle) \\
&+ \frac{1}{2\rho_B}(m_\sigma^2 \sigma^2 - m_\omega^2 \omega^2),
\end{aligned} \tag{7}
$$

where k_F is the Fermi momentum, and m_σ and m_ω are respectively the σ and ω meson masses. The ω field created by the uniformly distributed nucleons is determined by baryon

number conservation: $\omega = 3g_\omega^q \rho_B / m_\omega^2 = g_\omega \rho_B / m_\omega^2$ (where $g_\omega = 3g_\omega^q$), while the σ field is given by the thermodynamic condition: $(\partial E_{tot} / \partial \sigma) = 0$. This gives the self-consistency condition (SCC) for the σ field [7]:

$$\sigma = -\frac{4}{(2\pi)^3 m_\sigma^2} \left(\frac{\partial M^\star}{\partial \sigma} \right) \int^{k_F} d\vec{k} \frac{M^\star}{\sqrt{M^{\star 2} + \vec{k}^2}}, \tag{8}$$

where

$$\left(\frac{\partial M^\star}{\partial \sigma} \right) = -3g_\sigma^q S_N(\sigma) = -g_\sigma C_N(\sigma). \tag{9}$$

Here $g_\sigma = 3g_\sigma^q S_N(0)$ and $C_N(\sigma) = S_N(\sigma)/S_N(0)$, with the quark scalar charge defined by $S_N(\sigma) = \int_{bag} d\vec{r} \, \bar{\psi}_q \psi_q$. We should note that because the scalar-type correlation does not directly involve the σ field the SCC is not modified by it. However, the correlations do affect the σ field through the quark wave function.

In actual calculations, the quark density, $\langle \psi_q^\dagger \psi_q \rangle$, in the total energy, Eq.(7), may be replaced by $3\rho_B$, and the quark scalar density, contributing to the effective quark mass, $m_q^\star$, is approximately given as $\langle \bar{\psi}_q \psi_q \rangle = (m_\sigma^2 / g_\sigma)\sigma$ because of the SCC (see also Ref.[6]).

Now we present the numerical results. First, we choose $m_q = 5$ MeV and the bag radius of the nucleon in free space, R_0, to be 0.8 fm. We calculate the matter propereties using not only QMC but also MQMC. For the latter we take a simple variation of the bag constant in the medium to illustrate the role of the short-range correlations: $(B/B_0)^{1/4} = \exp(-g_\sigma^B \sigma/M)$ with $g_\sigma^B = 2.8$ [8] and the bag constant in free space, B_0. In both models, the bag constant (in free space) and z parameter are determined to fit the free nucleon mass with $R_0 = 0.8$ fm. We find $B_0^{1/4} = 170.0$ MeV (in QMC, $B = B_0$ at all densities) and $z = 3.295$. The coupling constants, g_σ and g_ω, are determined so as to reproduce the binding energy $(-15.7$ MeV) at the saturation density $(\rho_0 = 0.15$ fm$^{-3})$. We find that $g_\sigma^2 = 67.80$ and $g_\omega^2 = 66.71$ for QMC and $g_\sigma^2 = 35.69$ and $g_\omega^2 = 80.68$ for MQMC. Note that the matter properties at ρ_0 in both models with short-range correlations are identical to those of the original models [7, 8]. This is because, in our simple geometric approach, the effect of nucleon overlap starts beyond ρ_0 (see below).

In Figs.1 and 2, we present the total energies per nucleon for QMC and MQMC,
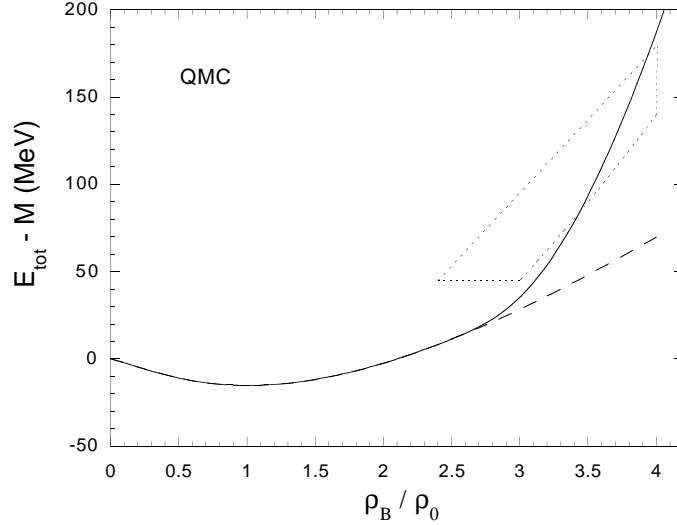
Figure 1: Energy per nucleon for symmetric nuclear matter. The dashed curve is the result of the original QMC. The solid curve is for the QMC with short-range correlations (QMCs). The region enclosed with the dotted curves is the empirical equation of state [9].

respectively. We determine the coupling constant, $\bar{f}_v^q$, so as to reproduce the empirical value of the energy around $\rho_B/\rho_0 = 2.5 \sim 4$ [9]. This yields the value $\bar{f}_v^q = 300$ for QMCs and $\bar{f}_v^q = 10$ for MQMCs. (Note that in MQMCs the overlap probability is much larger than that in QMCs at the same density because the bag radius in MQMC increases very rapidly at finite density. This is the reason why the strength of $\bar{f}_v^q$ for MQMCs is much weaker than that in QMCs.) For the strength of the scalar-type correlation, since we have no definite guideline, we take the same value in both models: $\bar{f}_s^q = 200$ (as an illustration). From the figures we can see that the nucleon overlap starts around $\rho_B/\rho_0 \sim 2.7$ or 1.3 for QMCs and MQMCs, respectively. The empirical energies at high densities (the region enclosed with the dotted curves in Figs. 1 and 2 [9]) are well reproduced in both models (in particular, in MQMCs) if the short-range correlations are considered.

In Fig.3, we show the change of the nucleon mass in matter. We can see that the effect of the short-range correlations on the mass is not strong. Correspondingly, the strength of the σ field in matter is not much altered by the correlations.
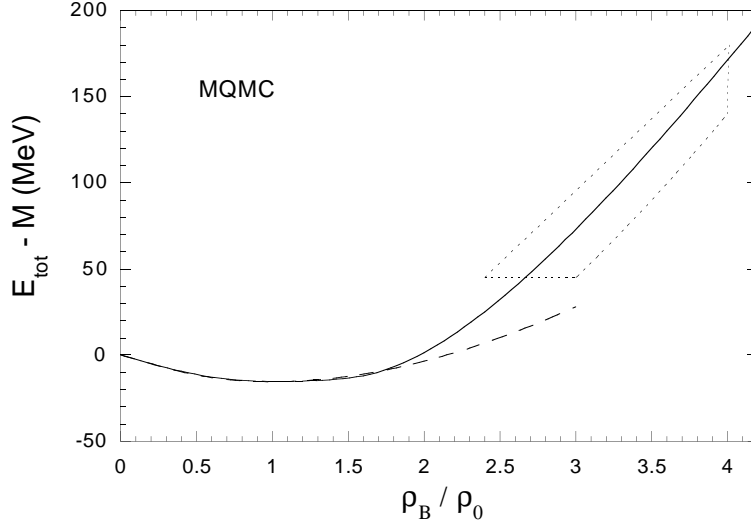
Figure 2: Same as in Fig.1 but for MQMC. The dashed curve is the result of the original MQMC. The solid curve is for the MQMC with short-range correlations (MQMCs). (In the original MQMC the solution of the quark eigenvalue beyond $\rho_B/\rho_0 \sim 3$ cannot be found [10].)

In Fig.4, we present the variation of the quark eigenvalue. In QMCs, the effect of the short-range correlations is weak, while in MQMCs the effect becomes very large as the nuclear density increases and the nucleons overlap more and more. Since we chose a repulsive scalar-type correlation, the effective quark mass in QMCs or MQMCs becomes larger than that in the original model as the density grows. This leads to a larger eigenvalue in QMCs or MQMCs. As a consequence of this repulsive correlation a solution for the quark eigenvalue in MQMCs can be found even beyond $\rho_B/\rho_0 \sim 3$.

Turning next to the size of the nucleon itself, as measured by the bag radius, we see in Fig.5 that the effect of the short-range correlations can be very significant. While the effect is small in QMCs, in MQMCs the bag radius starts to shrink as soon as the nucleons begin to overlap. We find a similar effect on the root mean square radius of the quark wave function. In the original MQMC it is well known that there is a serious problem concerning the bag radius. In particular, it grows rapidly at high density [3, 8, 10] because
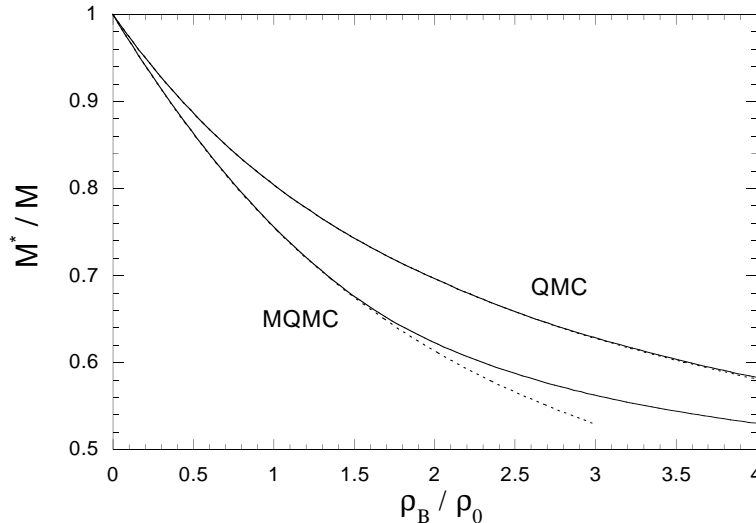
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Figure 3: Ratio of the effective nucleon mass to the free mass. The dotted curves are the results of the original QMC and MQMC. The solid curves are for MQMCs and MQMCs.

of the decrease of the bag constant. However, as we can see from the figure, the inclusion of a repulsive (scalar-type) short-range correlation yields a remarkable improvement for the in-medium nucleon size in MQMC.

In summary, we have studied (in mean-field approximation) the effect of short-range quark-quark correlations associated with nucleon overlap. We have found that the empirical equation of state at high density can be very well reproduced using a repulsive vector-type correlation. Furthermore, we have shown that a repulsive scalar-type correlation can counteract the tendency for the in-medium nucleon size to increase in MQMC. This may prove to be a significant improvement because there are fairly strong experimental constraints on the possible increase in nucleon size in-medium [8]. While our inclusion of correlations has been based on quite simple, geometrical considerations, in the future we would hope to formulate the problem in a more sophisticated, dynamical way [11] and to use it to study the properties of finite nuclei (including hyper nuclei [12]).
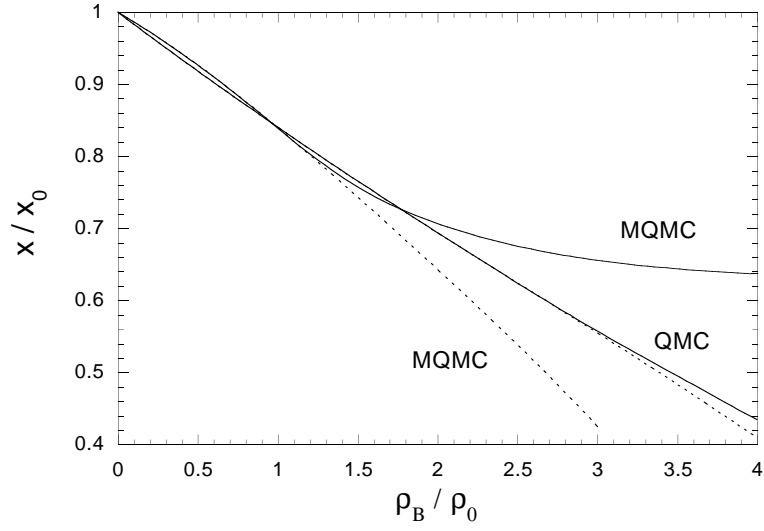
Figure 4: Ratio of the (lowest) quark eigenvalue in the matter to that in free space (x_0 = 2.052). The dotted curves are the results of the original QMC and MQMC. The solid curves are for QMCs and MQMCs.
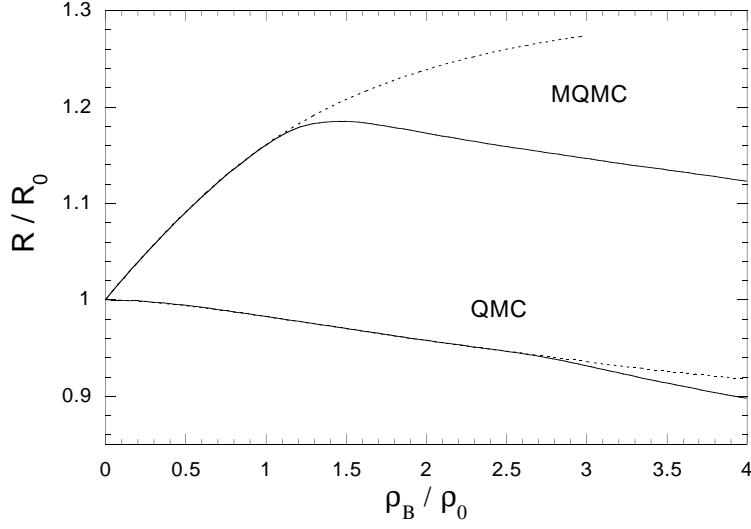
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Figure 5: Ratio of the in-medium bag radius of the nucleon to that in free space ($R_0 = 0.8$ fm). The dotted curves are the results of the original QMC and MQMC. The solid curves are for QMCs and MQMCs.

References

[1] P.A.M. Guichon, Phys. Lett. B 200 (1988) 235.

[2] A.W. Thomas, D.H. Lu, K. Tsushima, A.G. Williams, K. Saito, nucl-th/9807027, to appear in the proc. of the TJNAF users workshop, 15–18 June, 1998;
K. Saito, nucl-th/9811088, to appear in the proc. of the XIV international Seminar on High Energy Physics Problems, Dubna, 17–22 August, 1998.

[3] X. Jin, B.K. Jennings, Phys. Lett. B 374 (1996) 13; Phys. Rev. C 54 (1996) 1427.

[4] For example, C. DeTar, Phys. Rev. D 17 (1978) 323.

[5] F.E. Close, R.L. Jaffe, R.G. Roberts, G.G. Ross, Phys. Rev. D 31 (1985) 1004.

[6] P.G. Blunden, G.A. Miller, Phys. Rev. C 54 (1996) 359.

[7] P.A.M. Guichon, K. Saito, E. Rodionov, A.W. Thomas, Nucl. Phys. A 601 (1996) 349;
K. Saito, K. Tsushima, A.W. Thomas, Nucl. Phys. A 609 (1996) 339; Phys. Rev. C 55 (1997) 2637.

[8] D.H. Lu, K. Tsushima, A.G. Williams, A.W. Thomas, K. Saito, Nucl. Phys. A 634 (1998) 443.

[9] M. Sano, M. Gyulassy, M. Wakai, Y. Kitazoe, Phys. Lett. B 156 (1985) 27.

[10] H. Müller, B.K. Jennings, Nucl. Phys. A 626 (1997) 966; *ibid.* A 640 (1998) 55;
H. Müller, Phys. Rev. C 57 (1998) 1974.

[11] For the conventional nuclear physics, see C. Mahaux, R Sartor, Adv. Nucl. Phys. 20 (1991) 1.

[12] K. Tsushima, K. Saito, A.W. Thomas, Phys. Lett. B 411 (1997) 9; ibid. B 421 (1997) 413(E);

K. Tsushima, K. Saito, J. Haidenbauer, A.W. Thomas, Nucl. Phys. A 630 (1998) 691.